Supersymmetric branes in $AdS_2 \times S^2 \times CY_3$

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The problem of finding supersymmetric brane configurations in the near-horizon attractor geometry of a Calabi-Yau black hole with magnetic-electric charges (p^I, q_I) is considered. Half-BPS (Bogomol'nyi-Prasad-Sommerfield) configurations, which are static for some choice of global AdS_2 coordinate, are found for wrapped brane configurations with essentially any four-dimensional charges (u^I, v_I). Half-BPS multibrane configurations can also be found for any collection of wrapped branes provided they all have the same sign for the symplectic inner product $p^I v_I - u^I q_I$ of their charges with the black hole charges. This contrasts with the Minkowski problem for which a mutually preserved supersymmetry requires alignment of all the charge vectors. The radial position of the branes in global AdS_2 is determined by the phase of their central charge.

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I. INTRODUCTION

$AdS_2 \times S^2 \times CY_3$ flux compactifications of string theory arise as the near-horizon geometries of type IIA black holes. The fluxes are determined from the black hole charges. The vector moduli of the Calabi-Yau threefold and the radius of the $AdS_2 \times S^2$ are also determined in terms of these charges via the attractor equations [1,2]. These compactifications are interesting for several reasons. A central unsolved problem in string theory is to find—assuming it exists— a holographically dual CFT_1 for these compactifications.[1] Moreover recently a simple and unexpected connection was found between the partition function of the black hole and the topological string on the corresponding attractor Calabi-Yau [4]. In this paper we will further our understanding of these compactifications by analyzing the problem of supersymmetric brane configurations.

Following some review in section II, in section III the problem of supersymmetric branes is analyzed from the viewpoint of the four-dimensional effective $\mathcal{N} = 2$ theory on $AdS_2 \times S^2$. This analysis is facilitated by the construction [5] of the κ-symmetric superparticle action carrying general electric and magnetic charges (u^I, v_I) in such theories. It is found that there is always a supersymmetric trajectory whose position is determined by the phase of the central charge $Z(u^I, v_I)$. In global AdS_2 coordinates

$$ds^2 = R^2(-\cosh^2\chi d\tau^2 + d\chi^2 + d\theta^2 + \sin^2\theta d\phi^2) \quad (1.1)$$

the supersymmetric trajectory is at

$$\tanh\chi = \frac{\mathrm{Re}Z}{|Z|}. \quad (1.2)$$

For the general case $\chi \neq 0$ this trajectory is accelerated by the electromagnetic forces. We further consider n-particle configurations with differing charges and differing central charges Z_i, $i = 1, \ldots n$, constrained only by the condition that they all have the same sign for $\mathrm{Re}Z_i$. Surprisingly if the positions of the charges are each determined by (1.2), a common supersymmetry is preserved for the entire multiparticle configuration. This is quite different than the case of fluxless Calabi-Yau-Minkowski compactifications, where there is a common supersymmetry only if the charges are aligned. Supersymmetry preservation is possible only because of the enhanced near-horizon superconformal group. This phenomena should have a counterpart in higher AdS spaces and may be of interest for braneworld scenarios.

In section IV we consider the problem from the ten-dimensional perspective. For simplicity we consider only the $AdS_2 \times S^2 \times CY_3$ geometries arising from $D0 - D4$ Calabi-Yau black holes. Adapting the analysis of [6] to this context, we allow the wrapped branes to induce lower brane charges by turning on world-volume field strengths. We will find that there are no static, supersymmetric D0-branes in global coordinates because they want to accelerate off to the boundary of AdS_2 (there are static Bogomol'nyi-Prasad-Sommerfield (BPS) configurations in Poincaré coordinates). For a D2-brane embedded holomorphically in the Calabi-Yau, we will find that it is half-BPS and sits at $\chi = \tanh^{-1}(\sin\beta_{CY})$. Here, β_{CY} is related to the amount of magnetic flux on the world-volume. All D2-brane that are static with respect to a common global time in AdS_2 preserve the same set of half of the supersymmetries regardless of β_{CY}. Similar conclusions hold for D4, D6-branes wrapped on the Calabi-Yau. We also consider a D2-brane wrapped on the S^2 of the $AdS_2 \times S^2$ product and find that it is once again half-BPS and sits at $\chi = \tanh^{-1}(\sin\beta_{S^2})$.

A related problem is the case of supersymmetric multi-D0-brane configurations which generate higher brane charges via the Myers effect. This is considered in a companion paper [7].

[1]For some cases a dual CFT_2 is known [3].

II. PRELIMINARIES

In this section we briefly review some material which will be needed for our analysis. We are interested in type IIA string theory compactified on a Calabi-Yau 3-fold M, with 2-cycles labeled by α^A, where $A = 1, 2, \cdots, n \equiv h_{11}$. The low energy effective theory is $\mathcal{N} = 2$ supergravity coupled to n vector multiplets (and also $h_{21} + 1$ hypermultiplets which are not relevant in our discussion). This theory can be described using special geometry [8–12] and here we will follow the notation of [8]. The scalar components of the vector multiplets are described in terms of projective coordinates X^I, $I = 0, 1, \cdots, n$. The prepotential $F(X^I)$ is holomorphic and homogeneous of degree 2 in the X^I's. In the large volume limit F is of the form

$$F = D_{ABC}\frac{X^A X^B X^C}{X^0} + \cdots \tag{2.1}$$

where $D_{ABC} = -\frac{1}{6}C_{ABC}$, C_{ABC} being the triple intersection number of the 4-cycles dual to α^A, which we denote by Σ_A.

Extremal black holes of magnetic and electric charge $(p^0 = 0, p^A, q_0, q_A)$ are realized as a D4-brane wrapped on 4-cycle $P = \sum p^A \Sigma_A$ bound with q_0 D0-branes, together with q_A gauge field fluxes through the 2-cycles α^A. The asymptotic values of the moduli fields $X^I, F_I \equiv \partial_I F$ at infinity can be arbitrary. However at the black hole horizon they approach the fixed point values determined from the "attractor equations" [1,2]

$$p^I = \mathrm{Re}CX^I, \qquad q_I = \mathrm{Re}CF_I. \tag{2.2}$$

Using the tree level prepotential (2.1), the fixed points of the moduli are [13,14]

$$CX^0 = i\sqrt{\frac{D}{\hat{q}_0}}, \qquad CX^A = p^A + \frac{i}{6}\sqrt{\frac{D}{\hat{q}_0}}D^{AB}q_B \tag{2.3}$$

where

$$D \equiv D_{ABC}p^A p^B p^C, \tag{2.4}$$

$$\hat{q}_0 \equiv q_0 + \frac{1}{12}D^{AB}q_A q_B, \tag{2.5}$$

$$D_{AB} \equiv D_{ABC}p^C, \tag{2.6}$$

$$D^{AB}D_{BC} = \delta^A_C. \tag{2.7}$$

The near-horizon geometry of the 4D extremal black hole is $AdS_2 \times S^2$ with the moduli at their attractor values. We are interested in string theory on the global $AdS_2 \times S^2 \times M$ geometry. The radius R of AdS_2 and S^2, which is the same as the radius of the extremal black hole, is determined in terms of the charges (p^I, q_I) via

$$R = \sqrt{2}(D\hat{q}_0)^{1/4} \tag{2.8}$$

where hereafter we work mainly in four-dimensional Planck units.

The metric on the Poincaré patch of $AdS_2 \times S^2$ is

$$ds^2 = R^2\Big(\frac{-dt^2 + d\sigma^2}{\sigma^2} + d\theta^2 + \sin^2\theta d\phi^2\Big) \tag{2.9}$$

while the metric is

$$ds^2 = R^2(-\cosh^2\chi d\tau^2 + d\chi^2 + d\theta^2 + \sin^2\theta d\phi^2) \tag{2.10}$$

in global coordinates. In much of this paper we deal with the case $q_A = 0$, and here the Ramond-Ramond (RR) field strengths are

$$F_{(2)} = \frac{1}{R}\omega_{AdS_2}, \qquad F_{(4)} = \frac{1}{R}\omega_{S^2} \wedge J, \tag{2.11}$$

where $\omega_{AdS_2} = R^2\cosh\chi d\tau \wedge d\chi$ is the volume form on AdS_2, $\omega_{S^2} = R^2\sin\theta d\theta \wedge d\phi$ is the volume form on the S^2, and J is the Kähler form on the Calabi-Yau. In particular, the Kähler volume of the 2-cycles α^A are determined by the charges as

$$\frac{1}{2\pi\alpha'}\int_{\alpha^A} J = 2\pi p^A\sqrt{\frac{q_0}{D}}. \tag{2.12}$$

III. FOUR-DIMENSIONAL ANALYSIS

Flux compactifications on a Calabi-Yau threefold are described by an effective $d = 4$, $\mathcal{N} = 2$ supergravity with an $AdS_2 \times S^2$ vacuum solution whose moduli are at the attractor point with charges (p^I, q_I). This theory contains zerobranes[2] with essentially arbitrary charges (u^I, v_I) arising from various wrapped brane configurations. The κ-symmetric world line action of these zerobranes was determined in [5]. In this section we use the results of [5] to determine the possible supersymmetric world line trajectories.

The Killing spinor equation is

$$\nabla_\mu \epsilon_A - \frac{i}{2}\epsilon_{AB}T^-_{\mu\nu}\gamma^\nu\epsilon^B = 0, \tag{3.1}$$

where ϵ^A, $\epsilon_A = (\epsilon^A)^*$ $(A = 1, 2)$ are chiral and antichiral R-symmetry doublets of spinors. T^- is the anti-self-dual part of the graviphoton field strength, satisfying

$$Z_{BH} = \frac{1}{4\pi}\int_{S^2} T^- = e^{-\mathcal{K}/2}(F_I p^I - X^I q_I), \tag{3.2}$$

where $\mathcal{K} = -\ln i(\overline{X}^I F_I - X^I \overline{F}_I)$ is the Kähler potential. Define the phase of the central charge $e^{i\alpha} = Z_{BH}/|Z_{BH}|$. Then we can write $T^- = -ie^{i\alpha}(1 + i*)F$, where $F = \frac{1}{R}\omega_{AdS}$. In terms of the doublet of spinors (ϵ_1, ϵ^2) and (ϵ^1, ϵ_2), the Killing spinor equation can be written as

[2] We use the term zerobrane in a general sense and do not specifically refer here to a ten-dimensional D0-brane.

$$\nabla_\mu \epsilon + \frac{i}{2} e^{-i\alpha\gamma_5} \not{F} \gamma_\mu \sigma^2 \epsilon = 0. \tag{3.3}$$

Note that there is an ambiguity in choosing the overall phase of the moduli fields and the central charge,

$$X^I \to e^{i\theta} X^I, \qquad F_I \to e^{i\theta} F_I, \qquad \epsilon \to e^{(i/2)\theta\gamma_5}\epsilon, \tag{3.4}$$

so we are free to set $\alpha = 0$.

The solutions to the Killing spinor equation in global $AdS_2 \times S^2$ coordinates (2.10) are [15]

$$\epsilon = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\exp\left(\frac{i}{2}\tau\gamma^1\sigma^2\right)R(\theta,\phi)\epsilon_0 \tag{3.5}$$

$$R(\theta,\phi) \equiv \exp\left(-\frac{i}{2}(\theta - \pi/2)\gamma^{012}\sigma^2\right) \times \exp\left(-\frac{i}{2}\phi\gamma^{013}\sigma^2\right) \tag{3.6}$$

where ϵ_0 is a doublet of arbitrary constant spinors. Alternatively, in the Poincaré metric (2.9), the Killing spinors are [16]

$$\epsilon = \sigma^{-1/2}R(\theta,\phi)\epsilon_0^+ \quad \text{and} \quad \epsilon = (\sigma^{1/2} + i\sigma^{-1/2}t\gamma^1\sigma^2)R(\theta,\phi)\epsilon_0^-, \tag{3.7}$$

where $\epsilon_0^\pm$ are constant spinors satisfying $-i\gamma^0\sigma^2\epsilon_0^\pm = \pm\epsilon_0^\pm$, and $R(\theta,\phi)$ denotes the rotation on the S^2 as in (3.5). Note that γ^μ are the *normalized* gamma matrices in the corresponding frame.

The zerobrane action constructed in [5] has a local κ-symmetry parameterized by a four-dimensional spinor doublet κ_A on the world line. In addition the spacetime supersymmetries ϵ_A act nonlinearly in Goldstone mode on the world line fermions. In general [17], a brane configuration trajectory will preserve a spacetime supersymmetry generated by ϵ if the action on the world-volume fermions can be compensated for by a κ transformation. This condition can typically be written

$$(1 - \Gamma)\epsilon = 0 \tag{3.8}$$

where Γ is a matrix appearing in the κ-transformations. For the case at hand it follows from the results of [5] that the condition is

$$\epsilon_A + e^{i\varphi}\Gamma_{(0)}\epsilon_{AB}\epsilon^B = 0 \tag{3.9}$$

$$\epsilon^A + e^{-i\varphi}\Gamma_{(0)}\epsilon^{AB}\epsilon_B = 0 \tag{3.10}$$

where $\Gamma_{(0)}$ is the gamma matrix projected to the zerobrane world line, and $e^{i\varphi}$ is the phase of the central charge Z of the zerobrane,

$$Z = e^{-\mathcal{K}/2}(u^I F_I - v_I X^I) = e^{i\varphi}|Z|, \tag{3.11}$$

where (u^I, v_I) are its magnetic and electric charges. In terms of the spinor doublet, one can write (3.9) as

$$-ie^{-i\varphi\gamma_5}\Gamma_{(0)}\sigma^2\epsilon = \epsilon. \tag{3.12}$$

Let us solve the condition for (3.12) to hold along the world line of a zerobrane sitting at constant (χ, θ, ϕ). Writing the Killing spinor as

$$\epsilon = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\exp\left(\frac{i}{2}\tau\gamma^1\sigma^2\right)\epsilon_0' \tag{3.13}$$

where $\epsilon_0' = R(\theta,\phi)\epsilon_0$, it suffices to solve

$$-ie^{-i\varphi\gamma_5}\gamma^0\sigma^2\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\epsilon_0' = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\epsilon_0' \tag{3.14}$$

$$-ie^{-i\varphi\gamma_5}\gamma^0\sigma^2\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2\epsilon_0' = \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2\epsilon_0'. \tag{3.15}$$

Some straightforward algebra simplifies the above equations to

$$-i\gamma^0\sigma^2(\cos\varphi + i\cosh\chi\sin\varphi\gamma_5 + \sinh\chi\sin\varphi\gamma_5\gamma^0\sigma^2)\epsilon_0' = \epsilon_0' \tag{3.16}$$

$$i\gamma^0\sigma^2(\cos\varphi - i\cosh\chi\sin\varphi\gamma_5 + \sinh\chi\sin\varphi\gamma_5\gamma^0\sigma^2)\epsilon_0' = \epsilon_0'. \tag{3.17}$$

A solution exists only when

$$\tanh\chi = \cos\varphi, \tag{3.18}$$

and therefore $\cosh\chi\sin\varphi = \pm 1$. Correspondingly, the constraints on ϵ_0' become

$$\gamma_5\gamma^0\sigma^2\epsilon_0' = \mp\epsilon_0', \tag{3.19}$$

where the sign on the RHS depends on the sign of $\sin\varphi$. This may be written as a condition on ϵ_0,

$$(1 \pm e^{(i/2)\phi\gamma^{013}\sigma^2}e^{i(\theta-\pi/2)\gamma^{012}\sigma^2}e^{(i/2)\phi\gamma^{013}\sigma^2}\gamma_5\gamma^0\sigma^2)\epsilon_0 = 0, \tag{3.20}$$

which makes it clear that zerobranes sitting at antipodal points on the S^2 will preserve opposite halves of the spacetime supersymmetries.

We conclude that a zerobrane following its charged trajectory in $AdS_2 \times S^2$ is half-BPS. The "extremal" case $\varphi = 0$ and π corresponds to the probe zerobrane with its charge aligned or antialigned with the charge of the original black hole. They cannot be stationary with respect to global time in the AdS_2. Using the Killing spinors on the Poincaré patch (3.7), it is clear that the extremal zerobranes following their charged trajectories (static on the Poincaré patch) are also half-BPS. In the special case $\varphi = \pi/2$ in (3.18) the zerobrane moves along an uncharged trajectory and experiences no electromagnetic forces . This corresponds to the case when the zerobrane charge is orthogonal to all the black hole charges.

A somewhat surprising feature is that there are supersymmetric *multiparticle* configurations of zerobranes with *unaligned* charges. All "positively-charged" zerobranes with $0 < \varphi < \pi$ preserve the same set of half of the supersymmetries, and all "negatively-charged" zerobranes with $-\pi < \varphi < 0$ preserve the other set. Using the attractor equations the positive charge condition can be written in terms of the symplectic product of the black hole and zerobrane charges as

$$u^I q_I - p^I v_I > 0. \tag{3.21}$$

Given an arbitrary collection of zerobranes obeying (3.21) there is a half-BPS configuration with the position of each trajectory determined in terms of the charges of the zerobrane by (3.18). Of course, such a supersymmetric configuration of particles with unaligned charges is not possible in the full black hole geometry prior to taking the near horizon limit. The preserved supersymmetry is part of the enhanced near-horizon supergroup.

This result is consistent with the expectation from the BPS bound. The energy of a charged zerobrane sitting at position χ the AdS_2 is given by

$$\begin{aligned} H &= |Z| \cosh\chi - \frac{\mathrm{Re}(Z\bar{Z}_{BH})}{|Z_{BH}|} \sinh\chi \\ &= |Z|(\cosh\chi - \cos\varphi \sinh\chi). \end{aligned} \tag{3.22}$$

where the first term comes from the gravitational warping, and the second term comes from the coupling to the gauge field potential. At the stationary point $\tanh\chi = \cos\varphi$, the energy of the zerobrane is

$$|Z \sin\varphi| = \frac{|\mathrm{Im} Z\bar{Z}_{BH}|}{|Z_{BH}|}. \tag{3.23}$$

Therefore, as long as $\mathrm{Im}(Z\bar{Z}_{BH})$ is always positive (or negative), the BPS energy for multiple zerobranes is additive, in agreement with the supersymmetry analysis above.

IV. TEN-DIMENSIONAL ANALYSIS

In this section we analyze supersymmetric brane configurations from the point of view of the ten-dimensional IIA theory on $AdS_2 \times S^2 \times CY_3$. For simplicity we will focus on specific examples rather than the most general solution.

The extremal black hole in type IIA string theory compactified on a Calabi-Yau manifold M preserves four supersymmetries. After we take the near-horizon limit, the number of preserved supersymmetries doubles to eight. We consider a background with only D0 and D4-brane charges, i.e. $q_A = p^0 = 0$, so that according to the attractor equations there is no B-field. The RR field strengths in the resulting $AdS_2 \times S^2 \times M_6$ are given as in (2.11). As shown in Appendix A, the ten-dimensional Killing spinor doublet is of the form

$$\varepsilon_1 = \epsilon_1 \otimes \eta_+ + \epsilon^1 \otimes \eta_-, \tag{4.1}$$

$$\varepsilon_2 = \epsilon^2 \otimes \eta_+ + \epsilon_2 \otimes \eta_-, \tag{4.2}$$

where $\eta_+, \eta_- = \eta_+^*$ are the chiral and antichiral covariantly-constant spinors on M; $\epsilon_A = (\epsilon^A)^*$, $\epsilon^{1,2}$ are four-dimensional chiral spinors satisfying the four-dimensional Killing spinor equation

$$\nabla_\mu \epsilon_A + \frac{i}{2} \not{F}^{(2)} \gamma_\mu (\sigma^2)_{AB} \epsilon^B = 0. \tag{4.3}$$

This is the same equation as (3.3) with $\alpha = 0$, and the solutions are given by (3.5) and (3.7).

We want to find all the BPS configurations of D-branes that are wrapped on compact portions of our background, and are pointlike in the AdS_2. In order for the D-brane to be supersymmetric, we only need to check that the κ-symmetry constraint

$$\Gamma\varepsilon = \varepsilon \tag{4.4}$$

is satisfied, where ε is the Killing spinor corresponding to the unbroken supersymmetry (more precisely, the pullback onto the brane world-volume). The κ projection matrix is given by [18–21]

$$\begin{aligned} \Gamma = \frac{\sqrt{\det G}}{\sqrt{\det(G + \mathcal{F})}} \sum_n \frac{1}{2^n n!} \Gamma^{\hat{\mu}_1 \hat{\nu}_1 \cdots \hat{\mu}_n \hat{\nu}_n} \mathcal{F}_{\hat{\mu}_1 \hat{\nu}_1} \cdots \mathcal{F}_{\hat{\mu}_n \hat{\nu}_n} \\ \times \Gamma_{(10)}^{n+(p-2)/2} \Gamma_{(0)} \sigma^1, \end{aligned} \tag{4.5}$$

$$\Gamma_{(0)} = \frac{1}{(p+1)! \sqrt{\det G}} \epsilon^{\hat{\mu}_0 \cdots \hat{\mu}_p} \Gamma_{\hat{\mu}_0 \cdots \hat{\mu}_p}, \tag{4.6}$$

where the hatted indices label coordinates on the brane world-volume, G is the pullback of the spacetime metric, and $\mathcal{F} = F + f^*(B)$ (the B-field is zero in our discussion). See Appendix A for conventions on 10D gamma matrices. Unless otherwise noted we will work in global coordinates (2.10).

A. D0-brane

For a static D0-brane in global coordinates, we have $\Gamma_{(0)} = \gamma^0$. The κ-symmetry matrix is

$$\Gamma = \Gamma_{(10)} \gamma^0 \sigma^1 \tag{4.7}$$

Writing the doublet ε in terms of the 4-dimensional spinor doublet ϵ

$$\varepsilon = \epsilon \otimes \eta_+ + \epsilon^* \otimes \eta_-, \tag{4.8}$$

The matrix Γ acts on ε as $\gamma^0 \sigma^1 \sigma^3 = -i\gamma^0 \sigma^2$. The κ-symmetry constraint (4.4) becomes

$$(1 + i\gamma^0 \sigma^2)\epsilon = 0. \tag{4.9}$$

Using the explicit solutions of the Killing spinors in global

AdS (3.5), we see that (4.9) cannot be satisfied at all τ, so a D0-brane static in global AdS can never be BPS. This is of course expected since the charged trajectory cannot be static in global coordinates. On the other hand, using (3.7) we see that a D0-brane static with respect to the Poincaré time is always half-BPS, as expected.

B. D2 wrapped on Calabi-Yau, $F = 0$

Now let us consider a D2-brane wrapped on M and static in global $AdS_2 \times S^2$, without any world-volume gauge fields turned on. The κ-symmetry matrix is

$$\Gamma = \frac{1}{2\sqrt{\det' G}} \gamma^0 \epsilon^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} \sigma^1 \tag{4.10}$$

where $\det'$ takes the determinant of the spatial components of the world-volume metric. Acting on ε, we have

$$\Gamma_{\hat{a}\hat{b}}\varepsilon = \partial_{\hat{a}} X^I \partial_{\hat{b}} X^J \gamma_{IJ} \varepsilon \tag{4.11}$$

$$= 2\partial_{\hat{a}} X^i \partial_{\hat{b}} X^{\bar{j}} \gamma_{i\bar{j}} \varepsilon + \partial_{\hat{a}} X^i \partial_{\hat{b}} X^j \gamma_{ij} \varepsilon + \partial_{\hat{a}} X^{\bar{i}} \partial_{\hat{b}} X^{\bar{j}} \gamma_{\bar{i}\bar{j}} \varepsilon \tag{4.12}$$

$$\begin{aligned} &= 2\partial_{\hat{a}} X^i \partial_{\hat{b}} X^{\bar{j}} (-g_{i\bar{j}} \gamma_{(6)}) \varepsilon \\ &\quad + \left(\frac{1}{2} \partial_{\hat{a}} X^i \partial_{\hat{b}} X^j \Omega_{ijk} \epsilon \otimes \gamma^k \eta_- + c.c. \right). \end{aligned} \tag{4.13}$$

The κ-symmetry constraint $\Gamma\varepsilon = \varepsilon$ implies $\epsilon^{\hat{a}\hat{b}} \partial_{\hat{a}} X^i \partial_{\hat{b}} X^j \Omega_{ijk} = 0$, which means that the D2-brane must wrap a holomorphic 2-cycle. It then follows that Γ acts on ε as $\Gamma\varepsilon = i\gamma^0 \gamma_{(6)} \sigma^1 \varepsilon = \gamma_{(4)} \gamma^0 \sigma^2 \varepsilon$. Therefore (4.4) becomes

$$(1 - \gamma_{(4)} \gamma^0 \sigma^2) \epsilon = 0. \tag{4.14}$$

It is clear that the wrapped D2-brane sitting at $\chi = 0$ in AdS_2 is half-BPS. Note that the D2-brane without gauge field flux does not feel any force due to the RR fluxes ($q_A = 0$), so its stationary position is at the center of AdS_2.

C. D2 wrapped on Calabi-Yau, $F \neq 0$

With general world-volume gauge field strength F turned on, the matrix Γ is

$$\Gamma = \frac{1}{\sqrt{\det'(G+F)}} \left(1 + \frac{1}{2} \Gamma^{\hat{a}\hat{b}} F_{\hat{a}\hat{b}} \Gamma_{(10)} \right) \gamma^0 \left(\frac{1}{2} \epsilon^{\hat{c}\hat{d}} \Gamma_{\hat{c}\hat{d}} \right) \sigma^1 \tag{4.15}$$

An argument nearly identical to the one given in [6] shows that the supersymmetric D2-brane must wrap a holomorphic 2-cycle, and the gauge flux F satisfies

$$\frac{\sqrt{\det G}}{\sqrt{\det(G+F)}} (f^* J + iF) = e^{i\beta} \,\mathrm{vol}_2 \tag{4.16}$$

where vol_2 is the volume form on the D2-brane (which is just f^*J for a holomorphically wrapped brane), and β is a constant phase determined in terms of the D0-brane charge $2\pi n = 1/2\pi\alpha' \int F$ via

$$\frac{\tan\beta}{2\pi\alpha'} \int J = 2\pi n. \tag{4.17}$$

If the probe D2-brane is wrapped on the 2-cycle $[\Sigma_2] = n_A \alpha^A$, then using (2.12) we have

$$\tan\beta = \frac{n}{n_A p^A} \sqrt{\frac{D}{q_0}} \tag{4.18}$$

Note that from (4.16) we have $\cos\beta > 0$, since J is positive when restricted to holomorphic cycles. The κ-symmetry condition then becomes

$$(1 - e^{-i\beta\gamma_{(4)}} \gamma_{(4)} \gamma^0 \sigma^2) \epsilon = 0 \tag{4.19}$$

These is identical to (3.12) if we set $\varphi = \beta - \pi/2$. We can immediately read off the conditions for the static D2-brane to preserve supersymmetry when it sits at $\theta = \pi/2$, $\phi = 0$ in the S^2:

$$\begin{aligned} &\sin\beta = \tanh\chi, \qquad \cos\beta = \mathrm{sech}\chi, \\ &(1 - \gamma_{(4)} \gamma^0 \sigma^2) \epsilon_0 = 0. \end{aligned} \tag{4.20}$$

We see that for general $-\pi/2 < \beta < \pi/2$, the D2-brane sits at $\chi = \tanh^{-1}(\sin\beta)$ and is half-BPS. In fact they all preserve the same half supersymmetries, as discussed in section III. Anti-D2-branes with gauge field fluxes wrapped on holomorphic 2-cycles will preserve the other half supersymmetries.

D. Higher dimensional D-branes wrapped on the Calabi-Yau

Let us consider D4, D6-branes that are wrapped on the Calabi-Yau and sit at constant position in global $AdS_2 \times S^2$. We shall use a trick [21] to write the matrix Γ as

$$\Gamma = e^{-A/2} \Gamma_{(10)}^{(p-2)/2} \Gamma_{(0)} e^{A/2} \sigma^1 \tag{4.21}$$

where

$$A = -\frac{1}{2} Y_{\hat{a}\hat{b}} \Gamma^{\hat{a}\hat{b}} \Gamma_{(10)} \tag{4.22}$$

and $Y_{\hat{a}\hat{b}}$ is an antisymmetric matrix (analogous to the phase β in the previous subsection), related to the gauge field strength matrix $F_{\hat{a}\hat{b}}$ by

$$F = \tanh Y \tag{4.23}$$

By the same arguments as before, one can show that the BPS D-branes must wrap holomorphic cycles. Note that A acts on the Killing spinor ε as $A\varepsilon = -iY_{\hat{a}\hat{b}} (f^*J)^{\hat{a}\hat{b}} \gamma_{(4)} \varepsilon$, and $\Gamma_{(0)}$ acts as $\gamma^0 (i\gamma_{(6)})^{p/2}$ (see Appendix). Let us define $\beta = -Y_{\hat{a}\hat{b}} (f^*J)^{\hat{a}\hat{b}}$. The κ-symmetry constraint can be written as

$$\Gamma\varepsilon = e^{-i\beta\gamma_{(4)}/2}\Gamma_{(10)}^{(p-2)/2}\gamma^0(i\gamma_{(6)})^{p/2}e^{i\beta\gamma_{(4)}/2}\sigma^1\varepsilon = \varepsilon. \quad (4.24)$$

We can simplify this to

$$-ie^{-i(\beta-p\pi/4)\gamma_{(4)}}\gamma^0\sigma^2\epsilon = \epsilon. \quad (4.25)$$

This equation indeed agrees with (4.9) and (4.19) in the cases $p = 0, 2$. It is also identical to (3.12) provided we set $\varphi = \beta - p\pi/4$. So we conclude that a general Dp-brane (p even) wrapped on a holomorphic cycle in the Calabi-Yau, possibly with world-volume gauge fields turned on, static in the S^2 and following its charged trajectory in the AdS_2 is half-BPS. As in [6] there is a deformation of the supersymmetry condition on the world-volume gauge field F. In particular, the D-brane sits at $\tanh\chi = \cos(\beta - p\pi/4)$.

E. D2 wrapped on S^2, $F = 0$

Now let us turn to D2-branes wrapped on the S^2 appearing in the $AdS_2 \times S^2 \times M$ product. The κ-symmetry matrix is $\Gamma = \Gamma_{(0)}\sigma^1 = \gamma^{023}\sigma^1$. (4.4) can be written as

$$(1 - \gamma^{023}\sigma^1)\epsilon = 0. \quad (4.26)$$

Defining $R(\theta, \phi)$ to be the S^2-dependent factors in (3.5), this condition becomes

$$(1 - \gamma^{023}\sigma^1)\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta, \phi)\epsilon_0 = 0, \quad (4.27)$$

$$(1 - \gamma^{023}\sigma^1)\exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)\gamma^1\sigma^2R(\theta, \phi)\epsilon_0 = 0. \quad (4.28)$$

A little algebra reduces these to

$$\cosh\frac{\chi}{2}(1 - \gamma^{023}\sigma^1)R(\theta, \phi)\epsilon_0 = \sinh\frac{\chi}{2}(1 + \gamma^{023}\sigma^1)R(\theta, \phi)\epsilon_0 = 0. \quad (4.29)$$

The only way to satisfy both equations is to set $\chi = 0$. Since $\gamma^{023}\sigma^1$ commutes with $R(\theta, \phi)$, we end up with the condition

$$(1 - \gamma^{023}\sigma^1)\epsilon_0 = 0. \quad (4.30)$$

We conclude that the D2-brane sitting at the center of AdS and wrapped on the S^2 is half-BPS.

F. D2 wrapped on S^2, $F \neq 0$

With gauge field strength $F = f\omega_{S^2}$ turned on, the κ-symmetry matrix acts on ε as

$$\Gamma\varepsilon = \frac{\sqrt{\det G}}{\sqrt{\det(G+F)}}\left(1 + \frac{1}{2}\Gamma^{\hat{a}\hat{b}}F_{\hat{a}\hat{b}}\Gamma_{(10)}\right)\Gamma_{(0)}\sigma^1\varepsilon \quad (4.31)$$

$$= \frac{1}{\sqrt{1+f^2}}(1 + \gamma^{23}f\Gamma_{(10)})\gamma^{023}\sigma^1\varepsilon \quad (4.32)$$

$$= \exp(\beta\gamma^{23}\Gamma_{(10)})\gamma^{023}\sigma^1\varepsilon = \gamma^{023}\sigma^1\exp(\beta\gamma^{23}\sigma^3)\varepsilon, \quad (4.33)$$

where $f \equiv \tan\beta$ ($\cos\beta > 0$). The condition (4.4) then becomes

$$(1 - \cos\beta\gamma^{023}\sigma^1 - i\sin\beta\gamma^0\sigma^2) \times \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta, \phi)\epsilon_0 = 0, \quad (4.34)$$

$$(1 - \cos\beta\gamma^{023}\sigma^1 + i\sin\beta\gamma^0\sigma^2) \times \exp\left(\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta, \phi)\epsilon_0 = 0, \quad (4.35)$$

A little algebra yields

$$(1 + \sin\beta\coth\chi)\epsilon_0 = 0, \quad (4.36)$$

$$(1 + \gamma^{023}\sigma^1\cot\beta\sinh\chi)\epsilon_0 = 0. \quad (4.37)$$

This means that $\sin\beta = -\tanh\chi$. In particular β, hence f, is constant on the world-volume. The condition on ϵ_0 becomes

$$(1 - \gamma^{023}\sigma^1)\epsilon_0 = 0. \quad (4.38)$$

These D-brane configurations are again half-BPS.

G. D-branes wrapped on S^2 and the Calabi-Yau

In general for a Dp-branes wrapped on S^2 times some $(p-2)$-cycle in the Calabi-Yau, and static in global AdS_2, the matrix Γ is essentially the product of the piece on S^2 and the piece on Calabi-Yau,

$$\Gamma\varepsilon = \exp(-\beta_{S^2}\gamma^{23}\sigma^3)\exp(-i\beta_{CY}\gamma_{(4)}) \times (i\gamma_{(4)})^{(p-2)/2}\gamma^{023}\sigma^1\varepsilon \quad (4.39)$$

where β_{CY} and β_{S^2} are the phases related to the world-volume gauge flux along the Calabi-Yau and S^2 directions as before. Define $\varphi_{CY} = \beta_{CY} - (p-2)\pi/4$, $\varphi_{S^2} = \beta_{S^2} + \pi/2$. The κ-symmetry constraint can be written as

$$-i\exp(-\varphi_{S^2}\gamma^{23}\sigma^3 - i\varphi_{CY}\gamma_{(4)})\gamma^0\sigma^2\epsilon = \epsilon \quad (4.40)$$

This is equivalent to

$$[1 + i\exp(-\varphi_{S^2}\gamma^{23}\sigma^3 - i\varphi_{CY}\gamma_{(4)})\gamma^0\sigma^2] \times \exp\left(-\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta, \phi)\epsilon_0 = 0, \quad (4.41)$$

$$[1 - i\exp(\varphi_{S^2}\gamma^{23}\sigma^3 + i\varphi_{CY}\gamma_{(4)})\gamma^0\sigma^2] \times \exp\left(\frac{i}{2}\chi\gamma^0\sigma^2\right)R(\theta, \phi)\epsilon_0 = 0. \quad (4.42)$$

A little algebra yields

$$[\sinh\chi - \cosh\chi\cos(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY})]R(\theta,\phi)\epsilon_0 = 0, \tag{4.43}$$

$$[\cosh\chi - \sinh\chi\cos(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY}) \tag{4.44}$$

$$-\gamma^{023}\sigma^1\sin(\varphi_{S^2} - i\gamma_{(4)}\gamma^{23}\sigma^3\varphi_{CY})]R(\theta,\phi)\epsilon_0 = 0, \tag{4.45}$$

If φ_{CY} and φ_{S^2} are both nonzero, the first equation can be satisfied only if

$$i\gamma_{(4)}\gamma^{23}\sigma^3 R(\theta,\phi)\epsilon_0 = mR(\theta,\phi)\epsilon_0, \quad m = \pm 1. \tag{4.46}$$

However, since $\gamma_{(4)}\gamma^{23}\sigma^3$ does not commute with $R(\theta,\phi)$ at generic points on the S^2, (4.46) can never be satisfied. Therefore such wrapped D-branes cannot be BPS.

If $\varphi_{S^2} = 0$, $\varphi_{CY} \neq 0$, we have

$$\tanh\chi = \cos\varphi_{CY} \tag{4.47}$$

and

$$(1 - \gamma_{(4)}\gamma^0\sigma^2)R(\theta,\phi)\epsilon_0 = 0. \tag{4.48}$$

However, in this case again $\gamma_{(4)}\gamma^0\sigma^2$ does not commute with $R(\theta,\phi)$ for generic (θ,ϕ), and hence (4.48) has no solution.

If $\varphi_{S^2} \neq 0$, $\varphi_{CY} = 0$, we find

$$\tanh\chi = \cos\varphi_{S^2} \tag{4.49}$$

and the second equation in (4.43) becomes

$$(1 - \gamma^{023}\sigma^1)\epsilon_0 = 0 \tag{4.50}$$

We see that such D-branes are half-BPS.

So far we have neglected an important subtlety. For D4 or D6-branes wrapped on S^2 times some cycle in the Calabi-Yau, the RR flux $F_{(4)}$ induces couplings of gauge fields on the brane world-volume

$$\int_{D4} A \wedge F_{(4)}, \tag{4.51}$$

$$\int_{D6} A \wedge F \wedge F_{(4)}, \tag{4.52}$$

Since $F_{(4)} = \frac{1}{R}\omega_{S^2} \wedge J$, we see that for the D4-brane wrapped on $S^2 \times \Sigma_2$ ($[\Sigma_2] = n_A\alpha^A$), the RR flux induces an electric charge density on the brane world-volume, of total charge

$$Q = \frac{1}{2\pi g_s}\int_{S^2\times\Sigma_2} F_{(4)} = \sum n_A p^A \tag{4.53}$$

Since the world-volume is compact, the Gauss law constraint requires the total charge to vanish. So we cannot wrap only a single D4-brane on $S^2 \times \Sigma$. One must introduce fundamental strings ending on the brane to cancel the electric charges. We then have $\sum n_A p^A$ fundamental strings ending on the D4-brane, and runoff to the boundary of AdS. This is interpreted as a classical "baryon" in the dual CFT.

Similarly for the D6-brane wrapped on $S^2 \times \Sigma_4$, one would have nonzero total electric charge on the world-volume if $\int_{\Sigma_4} F \wedge J \neq 0$. This again corresponds to certain "baryons" in the dual CFT.

Finally, a D6-brane wrapped on $S^2 \times \Sigma_4$ with general gauge field flux in the S^2 is half-BPS, as shown in (4.49) and (4.50).

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APPENDIX: THE 10-DIMENSIONAL KILLING SPINORS

In order to write a ten-dimensional spinor as the tensor product of four-dimensional and internal (Calabi-Yau) spinors, it is necessary to work with a tensor product of Clifford algebras. Let Γ^M denote the ten-dimensional Clifford algebra matrices, with $M = 0, \ldots, 10$, $\mu = 0, \ldots, 3$, and $m = 4, \ldots, 9$. We can decompose the Γ^M into a tensor product of four and six-dimensional Clifford matrices, denoted by γ^μ and γ^m, as

$$\Gamma^\mu = \gamma^\mu \otimes 1, \tag{A1}$$

$$\Gamma^m = \gamma_{(4)} \otimes \gamma^m. \tag{A2}$$

Using a mostly-positive metric signature, the following matrices have the desired properties that they anticommute with the appropriate gamma matrices and square to one:

$$\Gamma_{(10)} = -\Gamma^{0123456789}, \tag{A3}$$

$$\gamma_{(4)} = i\gamma^{0123}, \tag{A4}$$

$$\gamma_{(6)} = i\gamma^{456789}. \tag{A5}$$

With these sign conventions, $\Gamma_{(10)}$ decomposes in the desired way as $\Gamma_{(10)} = \gamma_{(4)} \otimes \gamma_{(6)}$.

As an ansatz for the Killing spinors, we assume they take the form

$$\varepsilon_1 = \epsilon_1 \otimes \eta_+ + \epsilon^1 \otimes \eta_-, \qquad \varepsilon_2 = \epsilon^2 \otimes \eta_+ + \epsilon_2 \otimes \eta_-, \tag{A6}$$

where the ε's are 10D Majorana-Weyl spinors, the η's are 6D covariantly-constant Weyl spinors on the Calabi-Yau, and the ϵ's are 4D Majorana spinors. We use chiral notation in which the chirality of the spinor is denoted by the position of the R-symmetry index. In particular, $\epsilon(A) = \epsilon^A + \epsilon_A$ where $\gamma_{(4)}\epsilon^A = \epsilon^A$ and $\gamma_{(4)}\epsilon_A = -\epsilon_A$. Of course, there are no Majorana-Weyl spinors in 3 + 1 dimensions; the four-dimensional chiral projections are related by $\epsilon_A =$

ϵ^{A*}. For the six-dimensional Weyl spinors, we use the standard notation where $\gamma_{(6)}\eta_{\pm} = \pm\eta_{\pm}$. Since we will work with type IIA, the tensor products have been chosen such that the ten-dimensional spinors are of opposite chirality. In doublet notation,

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \end{pmatrix} \tag{A7}$$

$\Gamma_{(10)}\varepsilon$ can be written as $-\sigma^3\varepsilon$. In addition, the following identities for the spinors $\eta_{\pm}$ will be useful:

$$\gamma_{\bar{i}}\eta_+ = 0, \quad \gamma_{ijk}\eta_+ = \Omega_{ijk}\eta_-, \quad \gamma_{ij}\eta_+ = \frac{1}{2}\Omega_{ijk}\gamma^k\eta_-, \quad \gamma_{\bar{i}\bar{j}kl}\eta_+ = (g_{k\bar{j}}g_{l\bar{i}} - g_{k\bar{i}}g_{l\bar{j}})\eta_+, \tag{A8}$$

$$\gamma_i\eta_- = 0, \quad \gamma_{\bar{i}\bar{j}\bar{k}}\eta_- = \Omega_{\bar{i}\bar{j}\bar{k}}\eta_+, \quad \gamma_{\bar{i}\bar{j}}\eta_- = \frac{1}{2}\Omega_{\bar{i}\bar{j}\bar{k}}\gamma^{\bar{k}}\eta_+, \quad \gamma_{ij\bar{k}\bar{l}}\eta_- = (g_{\bar{k}j}g_{\bar{l}i} - g_{\bar{k}i}g_{\bar{l}j})\eta_-. \tag{A9}$$

Given these *Ansätze*, we want to check that the supersymmetry variations of the background vanish modulo conditions on the four-dimensional Majorana components of the Killing spinors. Since we work only with bosonic backgrounds, we need only check the variations of dilatino and gravitino.

The supersymmetry variation of the dilatino is [22]

$$\delta\lambda = \frac{1}{2}(3\not{F}_{(2)}i\sigma^2 + \not{F}_{(4)}\sigma^1)\varepsilon, \tag{A10}$$

where $F_{(2)} = \frac{1}{R}\omega_{\mathrm{AdS}_2}$ and $F_{(4)} = \frac{1}{R}\omega_{S^2} \wedge J$. Taking note of the fact that $g^{i\bar{j}}\gamma_{\bar{i}j}\eta_{\pm} = 3\gamma_{(6)}\eta_{\pm}$ and $\not{\omega}_{S^2} = -i\not{\omega}_{AdS_2}\gamma_{(4)}$, we find that

$$\not{F}_{(4)}\varepsilon = -3i\not{\omega}_{\mathrm{AdS}_2}\gamma_{(4)}\gamma_{(6)}\varepsilon = -3\not{F}_{(2)}\sigma^3\varepsilon. \tag{A11}$$

As a result, the dilatino variation vanishes automatically.

The gravitino variation is

$$\delta\psi_M = \nabla_M\varepsilon + \frac{1}{8}(\not{F}_{(2)}\Gamma_M i\sigma^2 + \not{F}_{(4)}\Gamma_M\sigma^1)\varepsilon = 0. \tag{A12}$$

When the free index is holomorphic in the Calabi-Yau, this reduces to the following condition:

$$(\not{F}_{(2)}\gamma_m i\sigma^2 + \not{F}_{(4)}\gamma_m\sigma^1)\varepsilon = 0. \tag{A13}$$

Using the fact that $g^{i\bar{j}}\gamma_{i\bar{j}}\gamma_m\eta_{\pm} = \gamma_m\gamma_{(6)}\eta_{\pm}$, we find that $\not{F}_{(4)}\gamma_m\varepsilon = -\not{F}_{(2)}\gamma_m\sigma^3\varepsilon$. This works similarly for an antiholomorphic index, so the gravitino variation is identically zero when the free index is in the Calabi-Yau.

When the gravitino equation has its free index in the $AdS_2 \times S^2$ space, the variation becomes

$$\delta\psi_\mu = \left[\nabla_\mu \pm \frac{1}{8}\gamma_\mu(\not{F}_{(2)}i\sigma^2 - \sigma^1\not{F}_{(4)})\right]\varepsilon = 0, \tag{A14}$$

where the $\pm$ is $+$ if μ is in the S^2 and $-$ if μ is in the AdS_2. Using the same identity used for the dilatino equation, we get

$$\delta\psi_\mu = \left[\nabla_\mu \pm \frac{i}{2}\gamma_\mu\not{F}_{(2)}\sigma^2\right]\varepsilon = \left[\nabla_\mu + \frac{i}{2}\not{F}_{(2)}\gamma_\mu\sigma^2\right]\varepsilon. \tag{A15}$$

Demanding that the terms linear in η_+ and linear in η_- must vanish separately, we get the 4D equations

$$\left[\nabla_\mu + \frac{i}{2}\not{F}_{(2)}\gamma_\mu\sigma^2\right]\epsilon = 0, \tag{A16}$$

where $\epsilon = \begin{pmatrix} \epsilon_1 \\ \epsilon^2 \end{pmatrix}$.

It is useful to derive the action of $\Gamma_{(0)} = \frac{1}{(p+1)!\sqrt{\det G}}\epsilon^{\hat{\mu}_0\cdots\hat{\mu}_p}\Gamma_{\hat{\mu}_0\cdots\hat{\mu}_p}$ on the $\eta_{\pm}$ which live on the world-volume of holomorphically wrapped D-branes (see (4.5)). For D0-branes we have simply $\Gamma_{(0)} = \gamma^0$. For D2-branes, we have

$$\Gamma_{(0)}\eta_{\pm} = \gamma^0\epsilon^{i\bar{j}}\gamma_{i\bar{j}}\eta_{\pm} = i\gamma^0\gamma_{(6)}\eta_{\pm} \tag{A17}$$

For D4-branes, we have

$$\Gamma_{(0)}\eta_{\pm} = \gamma^0\frac{1}{4}\epsilon^{i\bar{j}k\bar{l}}\gamma_{i\bar{j}k\bar{l}}\eta_{\pm} = -\gamma^0\eta_{\pm} \tag{A18}$$

where we used the last column of (A8). Finally for D6-branes, we have $\Gamma_{(0)} = -i\gamma^0\gamma_{(6)}$ using (A3). These formulas can be summarized as $\Gamma_{(0)}\varepsilon = \gamma^0(i\gamma_{(6)})^{p/2}\varepsilon$.

[1] S. Ferrara, R. Kallosh, and A. Strominger, Phys. Rev. D **52**, R5412 (1995).
[2] A. Strominger, Phys. Lett. B **383**, 39 (1996).
[3] J. M. Maldacena, A. Strominger, and E. Witten, J. High Energy Phys. 12 (1997) 002.
[4] H. Ooguri, A. Strominger, and C. Vafa, Phys. Rev. D **70**, 106007 (2004).
[5] M. Billo, S. Cacciatori, F. Denef, P. Fre, A. Van Proeyen, and D. Zanon, Classical Quantum Gravity **16**, 2335 (1999).
[6] M. Marino, R. Minasian, G. W. Moore, and A. Strominger, J. High Energy Phys. 01 (2000) 005.
[7] D. Gaiotto, A. Simons, A. Strominger, and X. Yin, hep-th/0412179.

[8] B. de Wit, P. G. Lauwers, and A. Van Proeyen, Nucl. Phys. **B255**, 569 (1985) .
[9] E. Cremmer, C. Kounnas, A. Van Proeyen, J. P. Derendinger, S. Ferrara, B. de Wit, and L. Girardello, Nucl. Phys. **B250**, 385 (1985).
[10] A. Strominger, Commun. Math. Phys. **133**, 163 (1990).
[11] L. Castellani, R. D'Auria, and S. Ferrara, Classical Quantum Gravity **7**, 1767 (1990).
[12] A. Ceresole, R. D'Auria, S. Ferrara, and A. Van Proeyen, Nucl. Phys. **B444**, 92 (1995).
[13] K. Behrndt, G. Lopes Cardoso, B. de Wit, R. Kallosh, D. Lust, and T. Mohaupt, Nucl. Phys. **B488**, 236 (1997).
[14] M. Shmakova, Phys. Rev. D **56**, R540 (1997).
[15] N. Alonso-Alberca, E. Lozano-Tellechea, and T. Ortin, Classical Quantum Gravity **19**, 6009 (2002).
[16] H. Lu, C. N. Pope, and J. Rahmfeld, J. Math. Phys. (N.Y.) **40**, 4518 (1999).
[17] K. Becker, M. Becker, and A. Strominger, Nucl. Phys. **B456**, 130 (1995).
[18] M. Cederwall, A. von Gussich, B. E. W. Nilsson, and A. Westerberg, Nucl. Phys. **B490**, 163 (1997).
[19] M. Aganagic, C. Popescu, and J. H. Schwarz, Phys. Lett. B **393**, 311 (1997).
[20] E. Bergshoeff and P. K. Townsend, Nucl. Phys. **B490**, 145 (1997).
[21] E. Bergshoeff, R. Kallosh, T. Ortin, and G. Papadopoulos, Nucl. Phys. **B502**, 149 (1997).
[22] I. Bena and R. Roiban, Phys. Rev. D **67**, 125014 (2003).